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1989 J. Phys.: Condens. Matter 1 7543

(http://iopscience.iop.org/0953-8984/1/41/005)

J. Phys.: Condens. Matter **1** (1989) 7543–7553. Printed in the UK

Localised acoustic waves associated with a planar defect in a superlattice

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Abstract. We present a theoretical study of the existence of shear horizontal vibrations localised at a planar defect in a superlattice. Such a defect, which breaks the translational symmetry along the axis of the superlattice, is formed by a layer whose elastic parameters and thickness are different from those of the normal layers; it may result from inhomogeneities in the properties of the perfect superlattice. We show that localised shear horizontal modes associated with the planar defect may exist in the gaps of the superlattice or even below the bottom of the bulk bands and may in this case extend down to $k_\parallel = 0$ ($k_\parallel$ is the wavevector parallel to the layers). The dispersion relations of these waves are discussed as functions of the relative parameters of the superlattice and the planar defect. These results are qualitatively compared with those obtained in other surface and interface problems in which the existence of shear horizontal excitations have been investigated.

1. Introduction

It is now well established that the investigation of phonons in heterostructures and superlattices can provide much information on the composition, period, interfaces, strain field and generally speaking on the quality of these systems [1]. One may also think about future applications in acoustic and acousto-optic devices. Much of the work devoted to vibrations in superlattices has been directed towards the study of infinite or semi-infinite superlattices. Among the recent works in the field of acoustic waves let us mention the investigation of the dispersion relations of bulk phonons (for a general direction of propagation [2–3]) and of surface phonons [4–6] whose eigenfunctions are exponentially decaying far from the surface of the superlattice. Different types of surface wave (Love, Rayleigh, Sezawa) have also been observed by Brillouin scattering and by surface acoustic techniques [7–15]. The study of these surface phonons provides complementary information on the acoustic properties of the superlattices, especially in the vicinity of the surface.

In addition to surfaces, other defects that break the translational invariance of the perfect superlattice also modify the vibrational properties and may give rise to localised modes inside the gaps. As a first step one can deal with the simplest systems in which the symmetry of translation parallel to the layers is still conserved, as for example the case of a superlattice deposited on a substrate [16] or of a planar defect in a superlattice. It is interesting from a fundamental point of view to search for the conditions under which one can find localised modes associated with these defects, whose frequencies may be situated below the bulk bands (as in homogeneous materials) or inside the gaps of the superlattice. In addition, the investigation of these waves provides a tool for the

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determination of the acoustic properties of slabs in confined geometries. The appearance of these localised modes influences the transmission coefficient of acoustic waves through the whole structure which can be measured; this effect may also be exploited in relation to possible acoustic devices.

In this paper we study the existence of localised shear horizontal modes associated with a planar defect in a superlattice. Though very simple because involving only one direction of vibration, the study of shear horizontal waves in superlattices may lead to the discovery of new types of modes. Indeed, we know [4] that, in contrast to the free surface of a homogeneous medium, the surface of a superlattice can support shear horizontal localised vibrations. Similarly, we have shown recently [16] that such waves may exist at the interface between a superlattice and a substrate, while the interface between two homogeneous media cannot support such localised vibrations. Let us also mention that the results obtained here can be transposed without difficulty to pure longitudinal waves propagating along the axis of the superlattice because these modes also involve one vibrational component.

The planar defect considered in this paper is obtained by assuming that one layer in the superlattice has different elastic parameters and thickness than the normal layers. Such a situation can be produced artificially or may originate from the presence of inhomogeneities in the superlattice, such as fluctuation in the thickness of one layer or mixing between two adjacent layers at an interface; these inhomogeneities can therefore induce localised modes and contribute to the density of states inside the gaps of the superlattice. The derivation of the dispersion relations of the localised waves is presented in § 2. In § 3, we discuss a few examples of the dispersion curves and also examine the long wavelength behaviour of these waves. The conclusions are given in § 4 where we also present a brief survey of other surface and interface problems in which localised shear horizontal waves may exist.

2. Calculation of the dispersion relations

In our calculation all media are assumed to be of hexagonal symmetry with isotropic (0001) interfaces. In this case the shear horizontal vibrations are decoupled from those polarised in the sagittal plane [6]. Each layer is characterised by its elastic constants $C_{44}^{(i)}$ and $C_{66}^{(i)}$, mass density $\rho^{(i)}$ and thickness $d^{(i)}$ ($i = 1, 2$ for the two constituents of the superlattice and $i = \mathrm{p}$ for the planar defect). However the dispersion relations obtained below can also be used [6] after a simple modification for a superlattice made of cubic materials with (001) interfaces if the wavevector $\boldsymbol{k}_\parallel$ (parallel to the layers) is directed along the [100] or [110] directions. Indeed, for these two orientations of the wavevector $\boldsymbol{k}_\parallel$ the equations of motion of the elasticity theory as well as the interface boundary conditions are formally similar for cubic and hexagonal materials and therefore it is possible to establish a correspondence between the parameters of each cubic crystal and those of a 'fictitious' hexagonal crystal [6]. When $\boldsymbol{k}_\parallel$ is parallel to [100] (or [110]) the elastic constants $\mathscr{C}_{44}$ and $\mathscr{C}_{66}$ of this fictitious material are given by $\mathscr{C}_{44} = C_{44}$, $\mathscr{C}_{66} = C_{44}$ (or $\mathscr{C}_{44} = C_{44}$, $\mathscr{C}_{66} = (C_{11} - C_{12})/2$).

Let x_3 be the axis of the superlattice and $\boldsymbol{k}_\parallel$ parallel to the x_1 axis. Then in the case of shear horizontal vibrations, the only non-zero component of the displacement field is u_2 such that

$$u_2(\boldsymbol{x}, t) = u_2(x_3) \exp[\mathrm{i}(k_\parallel x_1 - \omega t)] \tag{1}$$

where ω is the frequency of the wave. The equation of motion of the elasticity in the bulk of each material becomes

$$[(d^2/dx_3^2) - \alpha_i^2]\, u_2(x_3) = 0 \tag{2}$$

where

$$\alpha_i^2 = (C_{66}^{(i)}|k_\parallel^2| - \rho^{(i)}\omega^2)/C_{44}^{(i)} \qquad (i = 1, 2, \mathrm{p}). \tag{3}$$

Therefore within the planar defect the displacement field becomes

$$u_2^{(\mathrm{p})}(\boldsymbol{x}, t) = (A_\mathrm{p}\exp(-\alpha_p x_3) + B_\mathrm{p}\exp(\alpha_p x_3))\exp[\mathrm{i}(k_\parallel x_1 - \omega t)] \tag{4}$$

where A_p and B_p are two unknown multiplicative coefficients. Similarly in each layer of the superlattice the solution to the displacement can be written as [6]

$$u_2^{(n,i)}(\boldsymbol{x}, t) = [A^{(n,i)}\exp(-\alpha_i x_3^{(n,i)}) + B^{(n,i)}\exp(+\alpha_i x_3^{(n,i)})]\exp[\mathrm{i}(k_\parallel x_1 - \omega t)] \tag{5}$$

where the superscript (n, i) refers to the layer of type i $(i = 1, 2)$ belonging to the nth unit cell of the superlattice and $x_3^{(n,i)}$ is a local coordinate $(-d_i/2 \leqslant x_3^{(n,i)} \leqslant d_i/2)$ introduced for convenience. In the superlattice one can use the transfer-matrix method to relate the displacement field in succesive unit cells. Moreover, the Floquet theorem enables one to introduce a wavevector $\boldsymbol{k}_3$ such that

$$A^{(n,i)} \text{ (or } B^{(n,i)}) = A_i \text{ (or } B_i) \exp(\pm \mathrm{i}k_3 nD). \tag{6}$$

Here $D = d_1 + d_2$ is the period of the superlattice. Let us notice [6, 3] that the wave vector $\boldsymbol{k}_3$ is real (or complex) if the frequency ω belongs to a bulk band (or to a gap) of the superlattice. In addition, the coefficients A_i and B_i can be determined, except for a multiplicative factor, by using the continuity of the displacements and of the normal stresses at two consecutive interfaces (see [4] or [6] for details). One can also derive the usual dispersion relation of bulk waves in the superlattice [4, 6]

$$\cos k_3 D = C_1 C_2 + \tfrac{1}{2}(F + 1/F)S_1 S_2 \tag{7}$$

where

$$C_i = \cosh(\alpha_i d_i) \qquad S_i = \sinh(\alpha_i d_i) \tag{8a}$$

and

$$F = C_{44}^{(1)}\alpha_1 / C_{44}^{(2)}\alpha_2. \tag{8b}$$

A wave localised at the planar defect must decrease exponentially on both sides of the defect, when penetrating deep into the superlattice. Thus the frequency ω of such a mode necessarily belongs to a gap. In each half of the superlattice, occupying respectively the x_3-positive and x_3-negative half-spaces, one can write the displacement field as in (5) and (6) with $\boldsymbol{k}_3$ complex. However, in each solution we should only keep one of the terms in $\exp(\pm \mathrm{i}k_3 nD)$ (6), i.e. the one which is decaying in that part of the superlattice. Therefore the displacement field behaves for example like $\exp(\mathrm{i}k_3 nD)$ in the part of the superlattice occupying the x_3-positive half-space and like $\exp(-\mathrm{i}k_3 nD)$ in the other part.

Now, the solution for each half of the superlattice is known except for a multiplicative factor, let us say A_+ and A_- respectively. Therefore we are left with four unknown coefficients $A_\mathrm{p}, B_\mathrm{p}, A_+$ and A_-. These coefficients satisfy a set of four linear homogeneous equations resulting from the boundary conditions at the interfaces between the planar defect and the superlattice. By putting the determinant of this set of equations equal to zero one obtains the dispersion relation of the localised waves at the planar defect:

(i)
$$(F - F^{-1})(1 - F_p^2)S_2S_p + [2S_1C_2 + (F + F^{-1})C_1S_2] \times [(1 + F_p^2)C_1S_p + 2F_pS_1C_p] - 2\mathrm{i}\sin(k_3D) \times [(1 + F_p^2)S_1S_p + 2F_pC_1C_p] = 0 \tag{9a}$$

when the planar defect is bounded on both sides by the material 1 of the superlattice;

(ii)
$$(FF_p^{-1} + F^{-1}F_p)C_1S_2S_p + (F_p + F_p^{-1})S_1C_2S_p - 2\mathrm{i}\sin(k_3D)C_p = 0 \tag{9b}$$

when the planar defect is bounded by material 1 on one side and by material 2 on the other side.

In these expressions C_i, S_i ($i = 1, 2$ or p) and F are defined as in (8) and

$$F_p = C_{44}^{(1)}\alpha_1/C_{44}^{(p)}\alpha_p. \tag{10}$$

Let us notice that by taking the particular limit of (9b) in which d_1 and d_2 go to infinity, one can obtain the dispersion relation of the modes localised at a sandwich [17, 18] ABC (a material B ≡ p sandwiched between two semi-infinite materials A ≡ 1 and C ≡ 2)

$$(F_p + FF_p^{-1})S_p + (1 + F)C_p = 0. \tag{11}$$

The dispersion relations (9) will be discussed in the next section where in particular we examine the long wavelength behaviour of the localised modes.

3. Examples and discussion of the dispersion curves

In the first part of this section we present a few illustrations of the dispersion curves of the localised waves and discuss their behaviour as functions of the elastic parameters and thicknesses of the layers. In the second part we investigate analytically the existence and dispersion of these modes in the long-wavelength approximation.

Table 1. Elastic parameters of the materials Y and Dy. The elastic constants C_{66} are obtained as $(C_{11}-C_{12})/2$.

Type of layer	C_{11} (10^{10} N m^{-2})	C_{12} (10^{10} N m^{-2})	C_{44} (10^{10} N m^{-2})	ρ (Kg m^{-3})
Y	7.79	2.85	2.431	4450
Dy	7.31	2.53	2.40	8560

3.1. General results

The results discussed below refer to a Y–Dy superlattice [19] where the crystals are of hexagonal symmetry and the interfaces have the (0001) orientation. The elastic parameters involved in the calculations are listed in table 1 and we have assumed that $d_1 = d_2 = D/2$. A first example of the bulk bands and localised modes as functions of $\boldsymbol{k}_\parallel$ is given in figure 1. Here we used the dispersion relation (9a) by assuming that the

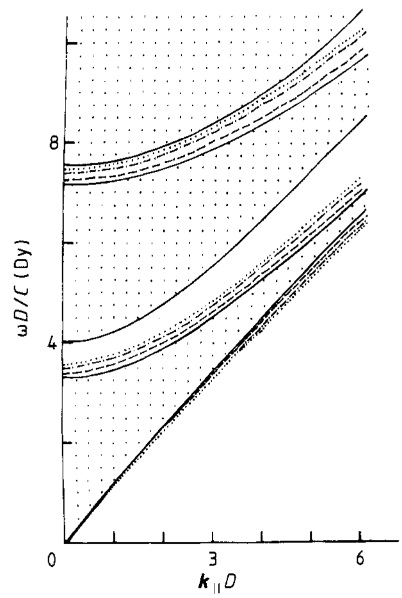


Figure 1. Bulk bands (shaded areas) and dispersion curves of localised modes at a planar defect in a Y–Dy superlattice. The defect is obtained by assuming that one layer of Y has a thickness d_p different from those of the normal layers. The localised branches are given for different values of $d_p/d(\mathrm{Y})$: 0.1 (. . . .); 0.2 (–·–); 0.5 (– – –). $C(\mathrm{Dy})$ is the velocity of sound in Dy given by $(C_{44}/e)^{1/2}$.

material 1 is Dy whereas materials 2 and p are both Y. This means that the only perturbation introduced in an otherwise perfect superlattice consists of a difference between the thickness d_p of one Y layer and those (d_2) of the normal Y layers. In figure 1 the dispersion curves are given for different values of the thickness d_p ranging from 0 to d_2. As a function of d_p, these curves are arranged in an increasing order inside the gaps whereas they are in a decreasing order below the bulk bands. At $d_2 = d_p$ one recovers the perfect superlattice and all the branches associated to the planar defect have merged into the bulk bands.

In order to show the variations of the localised modes at higher values of d_p $(d_p > d_2)$, we have presented in figure 2 the frequencies of these modes as functions of d_p at a given value of $k_\parallel$, namely $k_\parallel D = 0.5$. One can observe that by increasing d_p these frequencies are decreasing and the modes merge into the bulk bands, whereas new modes are extracted from higher bands. In addition, one can notice in figure 2 that at each frequency ω the localised mode is reproduced periodically as a function of d_p (the period being dependent upon ω). This behaviour can be explained in the following way: by fixing the values of $\boldsymbol{k}_\parallel$ and ω and by varying d_p, the only quantities that are varying in the dispersion equation (9*a*) are $C_p = \cosh(\alpha_p d_p)$ and $S_p = \sinh(\alpha_p d_p)$. In figure 2, the values of $\boldsymbol{k}_\parallel$ and ω are such that α_p is purely imaginary. Therefore C_p, S_p and the first member of (9*a*) become periodic functions of d_p with a period $2\pi/\alpha_p$. As in general the gaps are narrow in the superlattice, this period remains almost constant along a given gap. A result similar to that of figure 2 was obtained in our previous studies of the modes localised at the free surface of a superlattice [4–6], when varying the thickness of the layer which is at the surface. However, the periodic behaviour mentioned above has not been pointed out before.

In figure 2 we have also presented the results concerning a Dy–Y superlattice in which a layer of Dy has a different thickness than those of the normal Dy layers.

Figure 3 gives an illustration of the dispersion curves when the material p is different from both materials 1 and 2 which make the superlattice. The planar defect is assumed

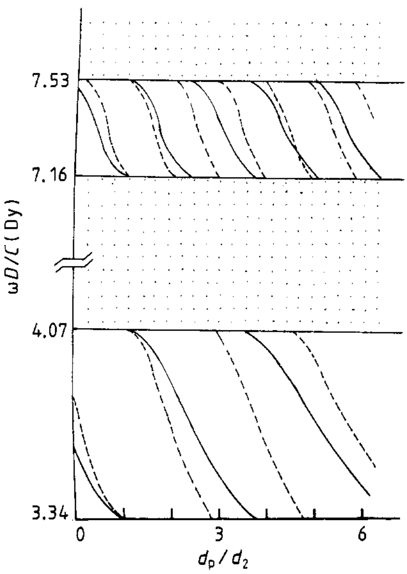


Figure 2. Full curves: variations of the frequencies of the localised waves presented in figure 1 as functions of the thickness d_p, at $k_{\|}D = 0.5$; the curves are presented in the first two gaps of the superlattice. Dashed curves: same as full curves but the planar defect is now obtained by assuming that one layer of Dy has a thickness d_p different from those of the normal layers.

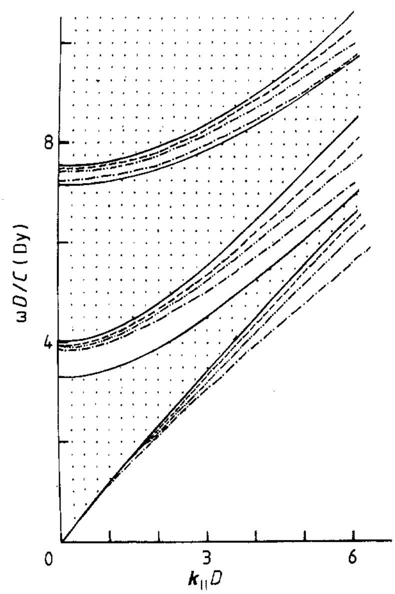


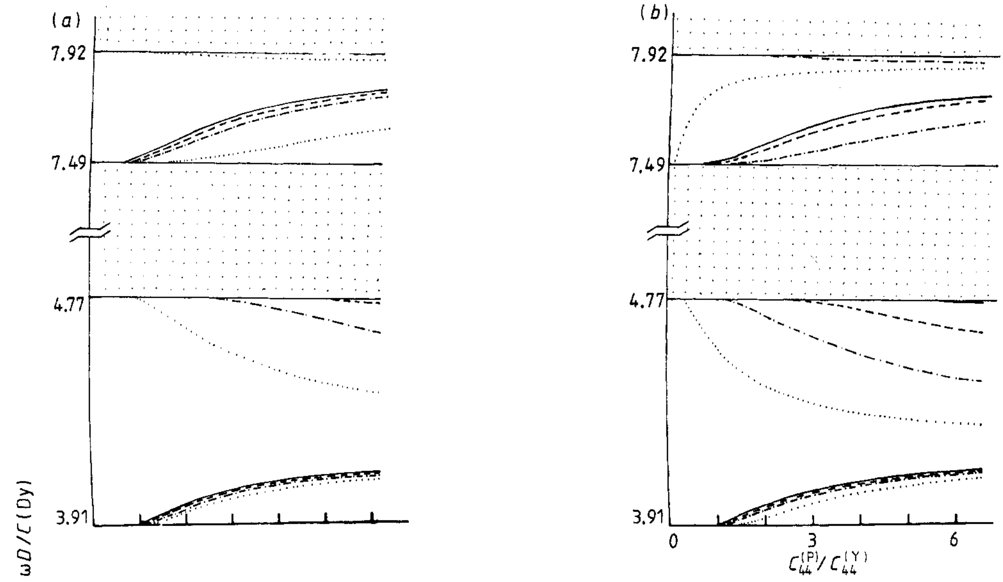
Figure 3. Bulk bands (shaded areas) and dispersion curves of localised modes in a Y–Dy superlattice when the planar defect p is made of a material different from the constituents of the superlattice. In this figure $d_p = d_1 = d_2$, $C_{66}^{(p)} = C_{66}^{(Y)}$, $\rho_p = (\rho^{(1)} + \rho^{(2)})/2$ and $C_{44}^{(p)}/C_{44}^{(Y)} =$ 0.5 (–·–) or 0.75 (–··–) or 1 (–––). The planar defect is bounded by Dy on both sides.

to be bounded by material 1 on both sides, so the dispersion relation (9*a*) has to be used. The localised modes are presented for a set of values of $C_{44}^{(p)}$, keeping constant the two other parameters $\rho^{(p)}$ and $C_{66}^{(p)}$ of the planar defect ($\rho^{(p)} = (\rho(\mathrm{Y}) + \rho(\mathrm{Dy}))/2$, $C_{66}^{(p)} = C_{66}^{(\mathrm{Y})}$). The localised branches are in an increasing order as functions of $C_{44}^{(p)}$, both inside the gaps and below the bulk bands. By increasing $C_{44}^{(p)}$ they merge into the bands, whereas new branches may be extracted from the bands, towards higher frequencies. In figure 4 we have kept $k_{\|}$ constant ($k_{\|}D = 2$) and presented the variations of the frequencies of the localised modes against $C_{44}^{(p)}$. A set of curves corresponding to different values of $C_{66}^{(p)}$ and $\rho^{(p)}$ are sketched. In fact these curves are labelled according to the two following parameters: $C_{66}^{(p)}/\rho^{(p)}$ which is the square velocity of sound in material p; and $C_{66}^{(p)}/C_{44}^{(p)}$ which defines the degree of elastic anisotropy in this material.

Finally we also considered examples in which the two layers in the superlattice have different thicknesses ($d_1 \neq d_2$). The qualitative conclusions are similar to the ones presented above but the quantitative results, and in particular the band gaps of the perfect superlattice, are totally different.

3.2. Long-wavelength approximation

We now investigate the existence and behaviour of the localised wave in the limit of long wavelengths compared to the thicknesses of the layers and in particular to the period D of the superlattice: this means that we assume $k_{\|}D \ll 1$ and $(C_{44}^{(i)}/\rho^{(i)})^{1/2}\omega/D \ll 1$.



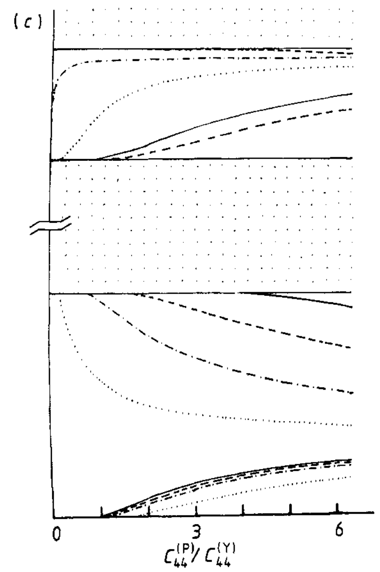


Figure 4. Variations of the frequencies of the localised waves as functions of $C_{44}^{(p)}$, at $k_{\|}D = 2$, in the first two gaps of the superlattice. In these figures $d_p = d_1 = d_2$, and the planar defect is bounded by Dy on both sides. The figures (*a*), (*b*), (*c*) correspond to different values of the anisotropy parameter $(C_{66}^{(p)}/C_{44}^{(p)})$, namely 0.5, 1 and 1.5 respectively in units of $(C_{66}^{(Y)}/C_{44}^{(Y)})$. Each figure contains a set of curves corresponding to different velocities of sound in material p; $(C_{66}^{(p)}/\rho^{(p)})$ takes the values 0.5 (····), 1 (–·–), 1.5 (– – –) and 2 (——) in units of $C_{66}^{(Y)}/\rho(Y)$.

Let us first notice that a Taylor expansion of (7) in this approximation gives the dispersion of the bottom of the bulk bands in the superlattice [6]

$$\omega = c_b k_{\|}[1 - \varepsilon_b |k_{\|}|^2 D^2 + o(|k_{\|}|^2 D^2)] \tag{12}$$

c_b is an average velocity of sound in the superlattice

$$c_b^2 = \bar{C}_{66}/\bar{\rho} \tag{13a}$$

and ε_b, which characterises the curvature of the dispersion curve, is given by

$$\varepsilon_b = [(d_1 d_2)^2 (\rho^{(1)}\rho^{(2)})^2 / 12 D^2 \bar{C}_{66} C_{44}^{e} \bar{\rho}^2](c_1^2 - c_2^2)^2 \tag{13b}$$

In (13) $\bar{C}_{66}$ and $\bar{\rho}$ are defined as arithmetic averages of the corresponding quantities in the two layers of the superlattice

$$\bar{C}_{66} = [d_1 C_{66}^{(1)} + d_2 C_{66}^{(2)}]/D \tag{14a}$$

$$\bar{\rho} = (d_1 \rho_1 + d_2 \rho_2)/D. \tag{14b}$$

C_{44}^{e} is an effective constant associated with the superlattice considered as an effective homogeneous medium [6], describing the propagation of shear horizontal waves along the axis of the superlattice

$$D/C_{44}^{\mathrm{e}} = d_1/C_{44}^{(1)} + d_2/C_{44}^{(2)}. \tag{14c}$$

Finally c_i ($i = 1, 2, \mathrm{p}$) is a velocity of sound in material i defined by

$$c_i = (C_{44}^{(i)}/\rho^{(i)})^{1/2}. \tag{14d}$$

Taylor expansions of (9) show that the dispersion relation of the localised mode below the bulk bands has to be searched under the form

$$\omega = c_{\mathrm{b}} \boldsymbol{k}_{\|}[1 - (\varepsilon_{\mathrm{b}} + \varepsilon_{\mathrm{p}})|\boldsymbol{k}_{\|}|^2 D^2 + \mathrm{o}(|\boldsymbol{k}_{\|}|^2 D^2)]. \tag{15}$$

This means that the velocity of the localised wave (when it exists) is equal to that of the bottom of the bulk bands (12), the only difference between these two curves coming from their curvatures.

Obviously, the dispersion curve (15) corresponds to a localised mode only if $\varepsilon_{\mathrm{p}} > 0$. However, the expansion of (9) leads to relations which give $\varepsilon_{\mathrm{p}}^{1/2}$, and not ε_{p}; therefore another condition is required in order that (15) represent a localised wave, namely the quantity giving $\varepsilon_{\mathrm{p}}^{1/2}$ should also be positive. We only give here the results of these calculations.

Let us first start from (9*b*) where the planar defect is bounded by materials 1 and 2. Then we obtain

$$\varepsilon_p = [(d_{\mathrm{p}})^2(\rho^{(\mathrm{p})})^2/4\bar{C}_{66}C_{44}^{\mathrm{e}}\bar{\rho}^2](c_{\mathrm{b}}^2 - c_{\mathrm{p}}^2)^2 - [(d_1 d_2)^2(\rho^{(1)}\rho^{(2)})^2/12D^2\bar{C}_{66}C_{44}^{\mathrm{e}}\bar{\rho}^2](c_1^2 - c_2^2)^2 \tag{16}$$

and we find that the localised wave (15) exists under the conditions

(i) $\quad c_{\mathrm{p}} < c_{\mathrm{b}}$ (17*a*)

(ii) $\quad \varepsilon_{\mathrm{p}} > 0.$ (17*b*)

The condition (17*a*) where $c_{\mathrm{p}} = (C_{44}^{(\mathrm{p})}/\rho^{(\mathrm{p})})^{1/2}$ is the velocity of sound in material p means that this material should be 'softer' than the superlattice considered as an effective homogeneous medium. On the other hand, the condition (17*b*) involves in a sophisticated expression the elastic parameters and thicknesses of the layers.

Let us now start from (9*a*). Here the perturbation with respect to the perfect superlattice consists of: one layer of material 1 (with a thickness d_1) plus one layer of material p (with a thickness d_{p}). One can define [20] a mean velocity of sound $\bar{c}_{\mathrm{p}}$ in this 'two-layer planar defect' as

$$\bar{c}_{\mathrm{p}}^2 = (d_1 C_{66}^{(1)} + d_{\mathrm{p}} C_{66}^{(\mathrm{p})})/(d_1 \rho^{(1)} + d_{\mathrm{p}} \rho^{(\mathrm{p})}). \tag{18}$$

Then the expansion of (9*a*) shows that the localised branch (15) exists if

(i) $\quad \bar{c}_{\mathrm{p}} < c_{\mathrm{b}}$ (19*a*)

(ii) $\quad \varepsilon_{\mathrm{p}} > 0$ (19*b*) where

$$\varepsilon_p = [(d_1 + d_p)^2/4\bar{C}_{66}C^e_{44}]\,\tilde{\rho}^2(c_b^2 - \tilde{c}_p^2)^2 - [(d_1d_2)^2(\rho^{(1)}\rho^{(2)})^2/12D^2\bar{C}_{66}C^e_{44}\bar{\rho}^2](c_1^2 - c_2^2)^2. \tag{20}$$

In this expression $\bar{C}_{66}$, C^e_{44}, $\bar{\rho}$, c_i, $\tilde{c}_p$ are defined as in (14) and (18) and

$$\tilde{\rho} = (d_1\rho^{(1)} + d_p\rho^{(p)})/(d_1 + d_p). \tag{21}$$

Again, the condition (19*a*) means that the velocity of sound in the planar defect should be smaller than a velocity in the superlattice considered as an effective medium.

Finally let us emphasise from (15) and (18)–(20) the particular case in which p ≡ 2: this means that the only perturbation in an otherwise perfect superlattice consists of a difference between the thickness of one layer of type 2 and those of the normal layers. In this case ε_P (20) becomes

$$\varepsilon_p = [(c_1^2 - c_2^2)^2(\rho^{(1)}\rho^{(2)})^2/4D^2\bar{C}_{66}C^e_{44}\bar{\rho}^2](d_1)^2[(d_p - d_2)^2 - (d_2)^2/3]. \tag{22}$$

Combining the conditions (19) one finds that the localised wave (15) exists if

(i) $c_1 < c_2$ and $d_p < d_2(1 - 3^{-1/2})$ (23*a*)

or

(ii) $c_1 > c_2$ and $d_p > d_2(1 + 3^{-1/2})$. (23*b*)

In figure 1 we have presented examples of the dispersion curves corresponding to three different values of d_p. For two of them the condition (23*a*) is satisfied and therefore the localised branch below the bulk bands extend down to $\boldsymbol{k}_\parallel = 0$; in the third case neither of the conditions (23) is satisfied and this localised wave merges into the bands at a finite value of $\boldsymbol{k}_\parallel$.

4. Concluding remarks

In this paper we obtained the dispersion relations and showed the existence of shear horizontal vibrations localised at a planar defect in a superlattice. The planar defect was defined as a layer having different elastic parameters and/or thickness than the normal layers in the superlattice. Obviously, more sophisticated situations in which the planar defect is an heterogeneous system composed of a few layers or/and is situated in the vicinity of the free surface of the superlattice may be imagined and studied by the same method as in this paper. Also, one can investigate the localised modes of sagittal polarisations, or, more generally, modes involving the three directions of vibration (see [3, 5, 6]) when the superlattice is composed of crystals of lower symmetry and/or the wave vector $\boldsymbol{k}_\parallel$ is along an arbitrary direction.

However the study of localised shear horizontal vibrations may lead to the finding of new types of modes because in the case of homogeneous materials such waves may exist near surfaces and interfaces only under special conditions. For example, the planar free surface of an homogeneous medium cannot support shear horizontal modes. The deposition of a thin layer of a softer material on this substrate gives rise to the well known Love modes [21], the first of which has the same phase velocity as in the substrate but a different curvature (see (15)). A surface shear horizontal branch is also obtained if the surface of the homogeneous medium is periodically rough instead of being planar [22].

This branch has the same behaviour as the first Love mode. Finally the free surface of a superlattice may support shear horizontal vibrations [4] whose frequencies are situated inside the gaps or possibly below the bottom of the bulk bands. When this last branch goes down to $k_{\|} = 0$ it also behaves like the first Love mode (see (15)).

The planar interface between two homogeneous media is also unable to support shear horizontal waves. However, such a mode can be induced by creating at the interface conditions which are the counterpart of those listed above for the surface problems. For example [17, 18], one can add a thin layer of another material between the two media in order to obtain a sandwich ABC (see the dispersion relation (11)); when A = C one obtains the particular case of a planar defect in medium A. An interface shear horizontal branch may also exist at a periodically rough interface between two media [23]; however, the amplitude of the roughness should now exceed a minimum value in order to extract such a branch from the bulk bands. Finally, interface shear horizontal waves can exist [16] at the boundary between a superlattice and a substrate which may be an homogeneous medium; their frequencies belong to the gaps of the superlattice or can possibly be below the bottom of the bulk bands. In contrast to the surface problems, in all these three examples an interface branch extracted below the bulk bands cannot extend down to $k_{\|} = 0$ but merges into these bands at a finite value of $k_{\|}$ (an exception occurs if the two media on either side of the interface have exactly the same phase velocities; this is, for example, the case for a planar defect ABA: a localised branch with a dispersion similar to that of the first Love mode appears [17] when the velocity of sound in medium B is lower than that in medium A).

The physical system considered in this paper is a generalisation of the simpler planar defect [17] ABA, where the medium A is now replaced by an heterogeneous material like a superlattice. This opens the possibility of having localised modes in the gaps of the superlattice and also extends the conditions to have a localised branch going down to $k_{\|} = 0$, below the bulk bands. With the present ability to manufacture heterostructures with a good precision, such a system can be created artificially. However, the presence of defects due to possible fluctuations in the elastic properties and thicknesses of the materials or due to the intermixing of two layers may also induce some modifications in the density of states. A further study should give these last variations not only in the gaps but also inside the bulk bands and especially indicate the appearance of resonance structures.

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